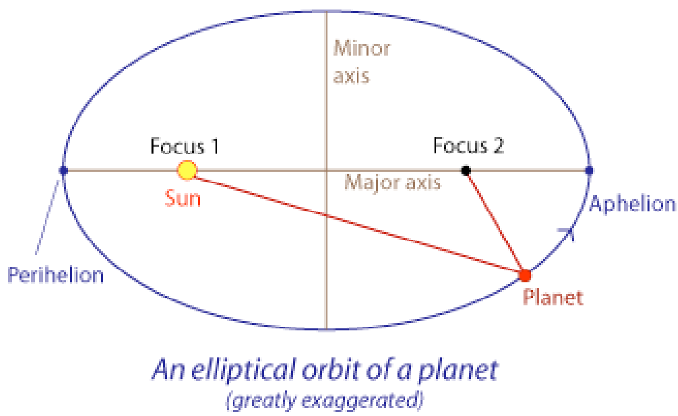
Earth Science: *Kepler's Laws of Planetary Motion*

Chapter 4-5: Planets, Minor Planets, Comets and more

Johannes Kepler formulated each of the laws presented here between 1609 and 1619 – and Danish astronomer Tycho Brahe gathered much of the information used in Kepler's early calculations in the years preceding the invention of the telescope.[1] It is amazing what these astronomers did without calculators, computers, and even without telescopes!

The Law of Ellipses: Kepler's 1st Law of Planetary Motion

Planets move around the Sun in ellipses, with the Sun at one focus.[1]



An ellipse is formed by two focus points – A circle has only one.

Image: http://outreach.atnf.csiro.au

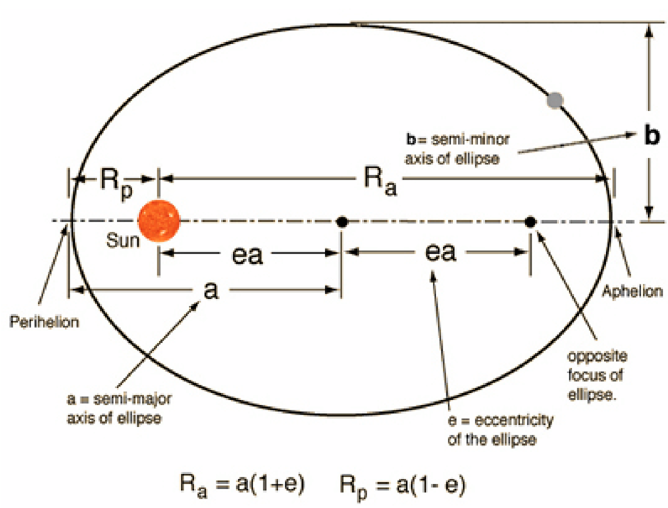


Diagram of an Elliptical Orbit: Mathematical Explanation for Kepler's 1st Law

Image: http://www.relativitycalculator.com/Kepler_1st_Law.shtml

Sources: [1]David P. Stern, http://www-istp.gsfc.nasa.gov/stargaze/Kep3laws.htm
[2]http://csep10.phys.utk.edu/astr161/lect/history/kepler.html
[3]http://www.aip.org/history/cosmology/ideas/kepler.htm

The Law of Equal Areas: Kepler's 2nd Law of Planetary Motion:

The line connecting the Sun to a planet sweeps equal areas in equal times. [1]

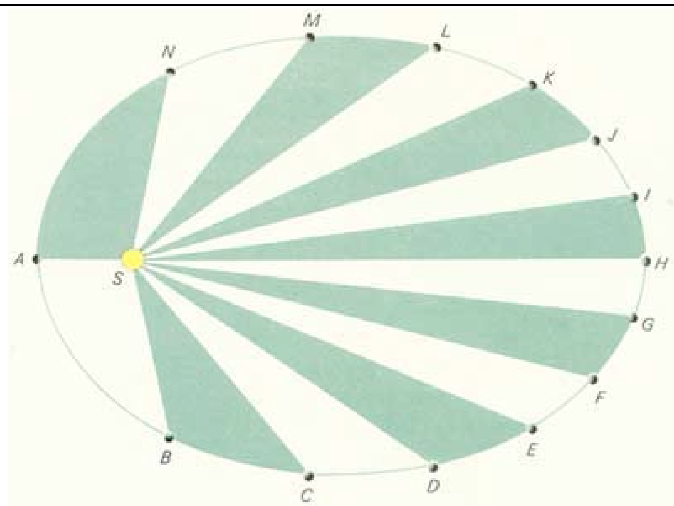


Diagram: Equal Areas in Equal Times

Image: http://www.aip.org/history/cosmology/ideas/kepler.htm

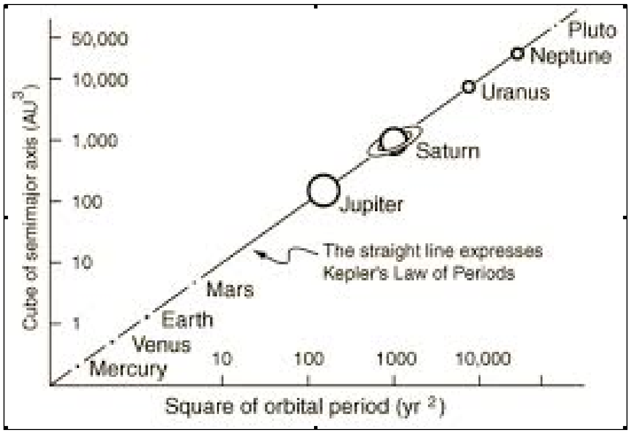
"A planet moves most rapidly on its elliptical orbit when it is at position A, nearest the focus of the ellilpse, S, where the sun is. The planet's orbital speed varies in such a way that in equal intervals of time it moves distances AB, BC, CD, and so on, so that regions swept out by the line connecting it and the sun (shaded and clear zones) are always the same in area." [3]

"The line joining the Sun and planet sweeps out equal areas in equal times, so the planet moves faster when it is nearer the Sun. Thus, a planet executes elliptical motion with constantly changing angular speed as it moves about its orbit. The point of nearest approach of the planet to the Sun is termed perihelion; the point of greatest separation is termed aphelion. Hence, by Kepler's second law, the planet moves fastest when it is near perihelion and slowest when it is near aphelion."[2]

Sources: [1]David P. Stern, http://www-istp.gsfc.nasa.gov/stargaze/Kep3laws.htm
[2]http://csep10.phys.utk.edu/astr161/lect/history/kepler.html
[3]http://www.aip.org/history/cosmology/ideas/kepler.htm

The Harmonic Law: Kepler's 3rd Law of Planetary Motion:

The square of the orbital period of a planet is proportional to the cube of the mean distance from the Sun (or in other words--of the"semi-major axis" of the ellipse - half the sum of smallest and greatest distance from the Sun)



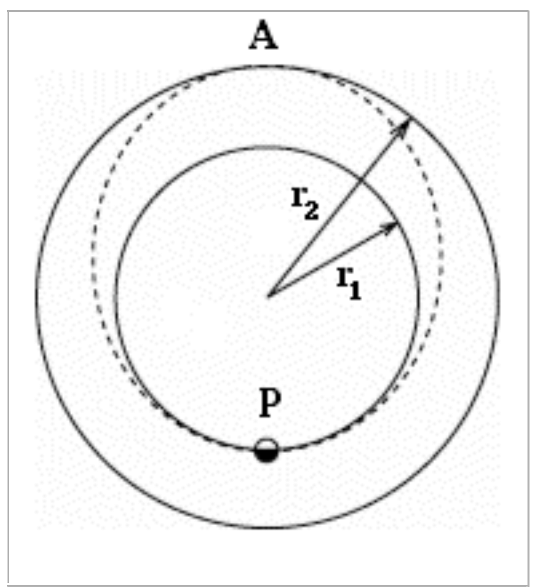
The farther a planet's orbit is from the Sun,
the longer its orbital period will be.

Image: http://hyperphysics.phy-astr.gsu.edu/hbase/astro/imgast/kep8.gif

Practical Applications of Kepler's 3rd Law:

All sorts of problems can be solved with Kepler's 3rd law. Here are a few: [1]

1. **How long does it take to reach Mars,** in the most efficient orbit? This is called the "Hohmann Transfer Orbit" (Wolfgang Hohmann, 1925). The spaceship must first get free of Earth (it still orbits the Sun together with Earth, at 30 km/s, at a distance of 1 AU), then it adds speed so that its aphelion (in its orbit around the Sun) just grazes the orbit of Mars, A = 1.524 AU (ignoring ellipticity).
2. For the Hohmann orbit, the smallest distance is 1.00 AU (Earth), the largest one 1.524 AU (Mars), so the semi-major axis is
3. **A = 0.5(1.00 + 1.524) = 1.262 AU**
4. **A^3 = 2.00992 = T^2**
5. The period is the square root **T = 1.412** years
   To reach Mars takes just half an orbit or
   T/2 = 0.7088 years = $\boxed{\textbf{~8.5 months}}$



The Hohmann Transfer Orbit

6. **How long would it take for a spacecraft from Earth to reach the Sun?**
   The Sun is the hardest object in the solar system to reach! It's far easier to escape to interstellar space (*yes, those people who speak of hurling nuclear waste into the Sun need to learn astronomy*.)

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[3]http://www.aip.org/history/cosmology/ideas/kepler.htm

To reach the Sun directly from Earth, we need shoot the spacecraft free of Earth. It still orbits the Sun with Earth, at 30 km/sec (low Earth orbit only takes 8 km/s), so we need give it an opposing thrust, adding (-30 km/s) to its velocity. It then falls straight into the Sun.

That orbit is also an ellipse, though a very skinny one. Its total length is 1 (AU), so the semimajor axis is A = 0.5 AU. By the 3rd law, $\mathbf{A^3 = 0.125 = T^2}$**,** and taking the square root , T=0.35355 years. We need divide this by 2 (it's a one-way trip!) and multiply by 365.25 to get days. Multiplying:

$$\mathbf{T/2 = (0.5)\ 0.35355\ (365.25) = \boxed{64.6\ days}}$$

7. **How far (from the center of Earth) do synchronous satellites orbit?** These are (mostly) communication satellites and have a 24 hour period, which helps them hang above the same station? The Moon is at 60 RE (earth radii) away and has a period of **T = 27.3217** days. The synchronous orbit is circular, so A is also its radius R. We get

$$\mathbf{(R/\ 60)^3 \ =\ R^3 / 216{,}000 \ =\ (1 / 27.3217\ days)^2}$$
$$\mathbf{=\ 1/\ (27.3217\ days)^2 \ =\ 1 / 746.5753}$$

so …

$$\mathbf{R^3 \ =\ 216{,}00/746.5753 \ =\ 289.32}$$

This number is between $6^3 = 216$ and $7^3 = 343$, so when the calculator gives R = 6.614 RE. you know you've got it about right.

8. **How far does Halley's comet go?**
Its period is about 75 years, and $75^2 = 5625$. Take the cube root: A = 17.784 AU. That, however is the SEMImajor axis. The length of the entire orbital ellipse is 2A = 35.57 AU. Perihelion is inside the Earth's orbit, less than 1 AU from the Sun, so aphelion is about 35 AU from the Sun--as the table shows, somewhere between Neptune's orbit and Pluto's

Sources: [1]David P. Stern, http://www-istp.gsfc.nasa.gov/stargaze/Kep3laws.htm
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[3]http://www.aip.org/history/cosmology/ideas/kepler.htm